Coherent Spin Ratchets

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We demonstrate that the combined effect of a spatially periodic potential, lateral confinement and spin-orbit interaction gives rise to a quantum ratchet mechanism for spin-polarized currents in two-dimensional coherent conductors. Upon external ac-driving, and in the absence of a static bias, the system generates a directed spin current while the total charge current is zero. We analyze the underlying mechanism by employing symmetry properties of the scattering matrix and numerically verify the effect for different setups of ballistic conductors. The spin current direction can be changed upon tuning the Fermi energy or the relative strength of Rashba and Dresselhaus spin-orbit coupling.

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Charge transport is usually studied by considering current in response to an externally applied bias. However, there has been growing interest throughout the last decade in mechanisms enabling directed particle motion in nanosystems without applying a net dc-bias. In this respect, ratchets, periodic structures with broken spatial symmetry, e.g. saw tooth-type potentials, represent a prominent class. Ratchets in the original sense are devices operating far from equilibrium by converting thermal fluctuations into directed particle transport in the presence of unbiased time-periodic driving [1]. First discovered in the context of (overdamped) classical Brownian motion [2, 3], the concept of dissipative ratchets was later generalized to the quantum realm [4]. More recently, coherent ratchets and rectifiers have gained increasing attention. They are characterized by coherent quantum dynamics in the central periodic system in between leads where dissipation takes place. Proposals comprise molecular wires [5] and cold atoms in optical lattices [6], besides Hamiltonian ratchets [7]. Experimentally, ratchet-induced charge flow in the coherent regime was first demonstrated for a periodic chain of triangular-shaped lateral quantum dots built from semiconductor heterostructures [8] and later in lateral superlattices [9].

Here we propose different class of ratchet devices, namely *spin ratchets* which act as sources for spin currents with simultaneously vanishing charge, respectively particle currents. To be definite we consider coherent transport through ballistic mesoscopic conductors in the presence of spin-orbit (SO) interaction. Contrary to ratchet mechanisms for directed particle motion, which rely on asymmetries in either the spatially periodic modulation or the time-periodic driving, a SO-based ratchet works even for symmetric (periodic) potentials. As possible realizations we have in mind semiconductor heterostructures with Dresselhaus [10] and Rashba [11] SO interaction. The latter can be tuned in strength by an external gate voltage thereby opening up the experimental possibility to control the spin evolution.

Among other features it is this property which is triggering recent broad interest in semiconductor-based spin electronics [12]. Also since direct spin injection from a ferromagnet into a semiconductor remains problematic [13], alternatively, several suggestions have been made for generating spin-polarized charge carriers without using magnets. In this respect, spin pumping appears promising, i.e. the generation of spin-polarized currents at zero bias via cyclic variation of at least two parameters. Different theoretical proposals based on SO [14] and Zeeman [15] mediated spin pumping in non-magnetic semiconductors have been put forward [16] and, in the latter case, experimentally observed in mesoscopic cavities [17].

While pumps and ratchets share the appealing property of generating directed flow without net bias, ratchet transport requires only a single driving parameter, the periodic ratchet potential has a strong collective effect on the spin current and gives rise to distinct features such as spin current reversals upon parameter changes.

Model and formalism.– We consider a two-dimensional coherent ballistic conductor in the plane (x, z) connected to two nonmagnetic leads. The Hamiltonian of the central system in presence of Rashba SO interaction reads

$$\mathcal{H}_{\rm c} = \frac{\hat{p}^2}{2m^*} + \frac{\hbar k_{\rm SO}}{m^*}(\hat{\sigma}_x \hat{p}_z - \hat{\sigma}_z \hat{p}_x) + U(x, z)\,. \quad (1)$$

Here m^* is the effective electron mass, $U(x, z)$ includes the ratchet potential in x- and a lateral transverse confinement in z-direction, and $\hat{\sigma}_i$ denote Pauli spin matrices. The effect of the SO coupling with strength $k_{\rm SO}$ is twofold: it is leading to spin precession and it is coupling transversal modes in the confining potential [18].

In view of a ratchet setup we consider an additional time-periodic driving term $\mathcal{H}_V(t)$ due to an external bias potential $V(t)$. The entire Hamiltonian then reads

$$\mathcal{H} = \mathcal{H}_{\rm c} + \mathcal{H}_V(t) \quad ; \quad \mathcal{H}_V(t) = -eV(t)g(x, z; V)\,, \quad (2)$$

where $g(x, z; V)$ describes the spatial distribution of the voltage drop and should in principle be self-consistently obtained from the particle density in the system.

We model spin-dependent transport within a scattering approach assuming that inelastic processes take place only in the reservoirs. Within this picture the probability amplitude for an electron to pass from the left to the right reservoir is given by the scattering matrix $\mathcal{S}_{n\sigma;n'\sigma'}(E,V)$, where the indices n' and n denote transverse modes and σ' and σ the spin directions in the left and right lead, respectively. The $\mathcal{S}$-matrix connects the current amplitudes $a_{n',\sigma'}$ and $b_{n,\sigma}$ of incoming and outgoing states:

$$b_{n,\sigma}(E,V) = \sum_{n'\in(L\cap R)} \sum_{\sigma'} \mathcal{S}_{n\sigma;n'\sigma'}(E,V)\; a_{n',\sigma'}(E)\,. \quad (3)$$

Ratchet mechanism: symmetry considerations.– To unravel the underlying mechanism for spin current generation we first analyze the symmetry properties of the Hamiltonian $\mathcal{H}$, Eq. (2), and of the $\mathcal{S}$-matrix with respect to time reversal and parity operations.

$\mathcal{H}$ is symmetrical under time-reversal, $\hat{\mathcal{K}}\mathcal{H}\hat{\mathcal{K}}^{-1} = \mathcal{H}$, where $\hat{\mathcal{K}} = -i\sigma_y\hat{\mathcal{C}}$ is the time reversal operator for a Pauli spinor, and $\hat{\mathcal{C}}$ causes complex conjugation. The action of $\hat{\mathcal{K}}$ on the asymptotic states is mainly to exchange incoming and outgoing waves. Furthermore, the spin is flipped ($\sigma \to -\sigma$), and a factor $\sigma = \pm 1$ arises: $\hat{\mathcal{K}}b_{n,\sigma} = \sigma a^*_{n,-\sigma}$. Applying $\hat{\mathcal{K}}$ to Eq. (3) and using the unitarity of the scattering matrix we obtain the following symmetry relation for the scattering amplitudes:

$$\mathcal{S}_{n,\sigma;n',\sigma'}(E,V) = \sigma\sigma'\mathcal{S}_{n',-\sigma';n,-\sigma}(E,V)\,. \quad (4)$$

We devise a minimum model for a spin ratchet mechanism by assuming that the leads are identical and that the potential $U(x,z)$ in Eq. (1) is spatially symmetric. Obviously, the total Hamiltonian (2) is then invariant under reflection at the y-axis, if the sign of the voltage in Eq. (2) is simultaneously changed. For a spinor wave function the reflection at the y-axis is generated by the operator $\hat{\mathcal{P}} = \exp[-i\pi\sigma_y/2]\hat{\mathcal{P}}_0(x,z) = -i\sigma_y\hat{\mathcal{P}}_0(x,z)$, where $\hat{\mathcal{P}}_0(x,z)$ inverts the x- and z-directions. Thus, the parity relation $\hat{\mathcal{P}}\mathcal{H}(V,t)\hat{\mathcal{P}}^{-1} = \mathcal{H}(-V,t)$ holds true.

The action of $\hat{\mathcal{P}}$ on the scattering states is to exchange the leads, i.e., a mode index n is replaced by its corresponding mode $\tilde{n}$. Moreover, the spin is flipped and factors σ for the spin and p_n for the parity of the mode n must be included. Under the action of $\hat{\mathcal{P}}$ incoming (outgoing) states stay incoming (outgoing) states. These considerations lead to the symmetry property

$$\mathcal{S}_{n,\sigma;m,\sigma'}(E,-V) = p_m p_n \sigma\sigma'\mathcal{S}_{\tilde{n},-\sigma;\tilde{m},-\sigma'}(E,+V)\,. \quad (5)$$

We proceed by specifying the external driving. We address the case of a rocking spin ratchet by choosing a time-periodic field with zero net bias. We focus on the case of an adiabatic driving field (such that the system can always adjust to the instantaneous equilibrium state), assuming that the ac-frequency is small compared to the relevant inverse time scales for the transmission. This is the common situation in corresponding experiments [8]. For convenience, we restrict the potential $V(t)$ to the values $\pm V_0$; generalizations to, e.g., harmonic driving are straight forward. The net current is then given by the average of the steady-state currents in the opposite rocking situations: $\langle I(V_0)\rangle = [I(+V_0) + I(-V_0)]/2$ which we compute within the Landauer formalism [19] relating conductances to transmission probabilities.

The averaged charge current $\langle I_{\mathrm{C}}\rangle$ and spin current $\langle I_{\mathrm{S}}\rangle$ can be expressed as the difference between charge (and spin) transmission probabilities $T^{\rightarrow}_{\mathrm{C/S}}$ from left to right (for bias $+V_0$) and $T^{\leftarrow}_{\mathrm{C/S}}$ from right to left (bias $-V_0$):

$$\langle I_{\mathrm{C/S}}(V_0)\rangle = \frac{e}{2h}\int_{E_{\mathrm{C}}}^{\infty} [f_{\mathrm{L}}(\epsilon,V_0) - f_{\mathrm{R}}(\epsilon,V_0)] \;\times \\ \times \left[T^{\rightarrow}_{\mathrm{C/S}}(\epsilon,V_0) - T^{\leftarrow}_{\mathrm{C/S}}(\epsilon,-V_0)\right] \mathrm{d}\epsilon\,. \quad (6)$$

Here, E_{C} is the energy of the conductance band edge, and $f_{\mathrm{L/R}}(\epsilon,V)$ are Fermi functions for which we use the property $f_{\mathrm{L}}(\epsilon,\pm V) = f_{\mathrm{R}}(\epsilon,\mp V)$. Introducing

$$T^{\rightarrow}_{\sigma,\sigma'}(E,V) = \sum_{n\in\mathrm{L},m\in\mathrm{R}} |\mathcal{S}_{m,\sigma;n,\sigma'}(E,V)|^2\,, \quad (7)$$

the transmission probabilities for charge and spin in Eq. (6) are defined as

$$T^{\rightarrow}_{\mathrm{C}}(E,V) = \sum_{\substack{\sigma'=\pm1\in\mathrm{L}\\ \sigma=\pm1\in\mathrm{R}}} T^{\rightarrow}_{\sigma,\sigma'}(E,V)\,, \quad (8)$$

$$T^{\rightarrow}_{\mathrm{S}}(E,V) = \sum_{\sigma'=\pm1\in\mathrm{L}} \left[T^{\rightarrow}_{+,\sigma'}(E,V) - T^{\rightarrow}_{-,\sigma'}(E,V)\right]. \quad (9)$$

The latter is given by the difference between the transmission of spin-up and spin-down electrons upon exit, with the spin measured with respect to the z-axis. Using the parity properties of the scattering matrix (5) in Eq. (8), we find for the transmission probabilities

$$T^{\rightarrow}_{\mathrm{C}}(E,V) = \sum_{\substack{m\in\mathrm{R}\\ \sigma=\pm1}} \sum_{\substack{n\in\mathrm{L}\\ \sigma'=\pm1}} |p_m p_n \sigma'\sigma\mathcal{S}_{\tilde{m},-\sigma;\tilde{n},-\sigma'}(E,-V)|^2 \\ = T^{\leftarrow}_{C}(E,-V)\,. \quad (10)$$

In view of Eq. (6), the ratchet charge current vanishes as expected: $\langle I_{\mathrm{C}}(V_0)\rangle = 0$. To evaluate the ratchet spin current we employ both, time-reversal (4) and parity (5) properties and find after a straight-forward calculation

$$T^{\rightarrow}_{\mathrm{S}}(E,V) - T^{\leftarrow}_{\mathrm{S}}(E,-V) = 2T^{\rightarrow}_{\mathrm{S}}(E,V)\,. \quad (11)$$

With Eq. (6) this implies the principle possibility for a spin current. The sign of Eq. (6) is indicating the flow direction. Here a few remarks are due:

(i) In contrast to the tunnel particle ratchet [4, 8] relying on a (finite) bias in the non-linear regime, the spin ratchet also operates in the linear response regime with a spin current $\langle I_{\mathrm{S}}(V_0)\rangle = (e^2/h)T^{\rightarrow}_{\mathrm{S}}(E,0)V_0$ (at $T=0$).

(ii) The spin-orbit ratchet mechanism stems from the spin inversion asymmetry of the Hamiltonian and does

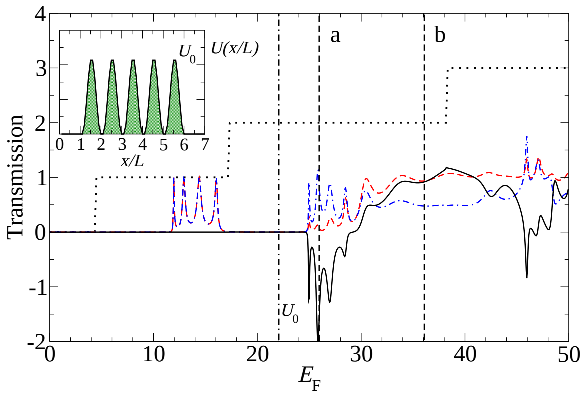

FIG. 1: (*Color online*) Spin-dependent transmissions as a function of the scaled injection (Fermi) energy E_F in the presence of Rashba spin-orbit interaction for a short periodic chain of five potential barriers (see inset, barrier height U_0 indicated as vertical dashed-dotted line in main panel). The dashed (red) trace shows T_{++} ('spin-up' to 'spin-up'), the dashed-dotted (blue) line T_{--} ('spin-down' to 'spin-down'), and the full line depicts the ratchet spin transmission (11). The sign of the latter is indicating the flow direction. For reference, the dotted staircase line shows the transmission ($T_{++} = T_{--}$) for a clean wire without potential barriers.

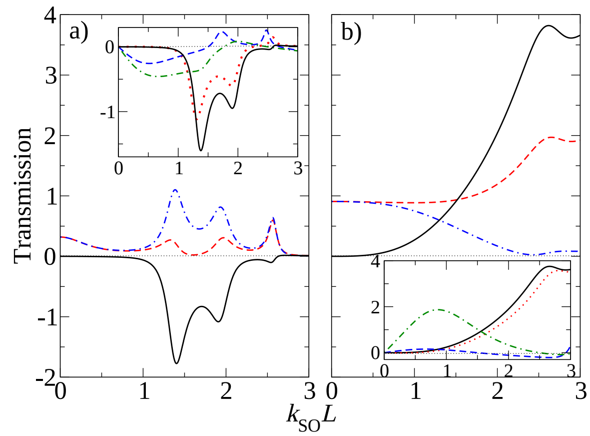

FIG. 2: (*Color online*) Transmissions for 5-barrier setup as a function of Rashba spin-orbit interaction $k_{SO}L$. Dashed (red) curve: T_{++}, dashed-dotted (blue): T_{--}, solid: ratchet spin transmission, Eq. (11). Panel (a): scaled energy $E_F = 26$ (vertical line a in Fig. 1), Panel (b): $E_F = 36$ (line b in Fig. 1). Insets: ratchet spin transmissions for different values of an additional Dresselhaus spin-orbit term $k_{SO}^D L = 0.25$ (solid line), 0.5 (dotted), 1.5 (dashed), 2.5 (dashed-dotted).

not require a broken spatial symmetry as usually do particle ratchets. For the case of a symmetric ratchet potential and negligible voltage drop, the spin flip transmissions $T_{\sigma,-\sigma}$ are identical: $T_{+-} = T_{-+}$, and hence the ratchet spin transmission (11) is given (up to a sign) by twice the difference between T_{++} and T_{--}.

(iii) To achieve $T_{++} \neq T_{--}$, at least two transverse modes should be available (which upon SO-induced hybridization give rise to finite spin-flip contributions T_{+-}, T_{-+} [18]), as we will see in the following case study.

Ratchet mechanism: numerical results.– We illustrate the predictions above by performing numerical calculations for the Hamiltonian (1,2). The $\mathcal{S}_{n\sigma';m\sigma}(E,V)$ are obtained by projecting the Green function of the open ratchet system onto an appropriate set of asymptotic spinors defining incoming and outgoing channels. For the efficient calculation of the $\mathcal{S}$-matrix elements a real-space discretization of the Schrödinger equation [19] combined with a recursive algorithm for the Green functions was implemented for spin-dependent transport [20, 21].

As a model for a spin ratchet we consider a ballistic two-dimensional quantum wire of width W with Rashba SO strength k_{SO} and a one-dimensional periodic modulation (period L) composed of a set of N symmetric potential barriers $U = U_0[1 - \cos(2\pi x/L)]$. To simplify the assessment of the rich parameter space (injection energy $E_F, U(x), V, W, k_{SO}, N$) of the problem ($L$ can be scaled out) and to analyse the mechanisms for spin currents, we first consider a strip with $N = 5$ symmetric potential barriers (see inset in Fig. 1) and few open transverse modes; furthermore we neglect the voltage drop [22]. Figure 1 shows (for $k_{SO}L = 1.5$) the numerically obtained transmission probabilities $T_{++}(E)$ (dashed line) and $T_{--}(E)$ (dashed-dotted). The solid line represents the resulting ratchet spin transmission $2[T_{++}(E) - T_{--}(E)]$ (Eq. (11) and item (ii)). For comparison, the dotted step function shows the successive opening of transverse modes $n = 1, 2, 3$ in the overall transmission of the conductor without potential barriers.

At energies below U_0 the transmissions T_{++} and T_{--} of the 5-barrier system are suppressed up to a sequence of four peaks representing resonant tunneling through states which can be viewed as precursors of the lowest Bloch band in the limit of an infinite periodic potential. Since only one transverse mode is open, there is no net spin transmission, $T_{++} = T_{--}$, see item (iii). Two related transmission peak sequences reappear at higher energies, owing to corresponding resonant states involving the second and third transverse mode. Due to SO-induced coupling and interference between different open modes these peaks are irregular. In particular, this mode mixing gives rise to the asymmetry $T_{++} \neq T_{--}$ and, as consequence, to the finite ratchet spin transmission (solid line). Figure 1 demonstrates moreover that the associated spin current changes sign several times upon variation of the energy, opening up the experimental possibility to control the spin current direction through the carrier density via an external gate. This energy dependence of the spin current implies also current inversion as a function of temperature [21]. Such behavior is considered as typical for quantum (particle) ratchets [4, 8].

In Fig. 2 we present the ratchet spin transmission,

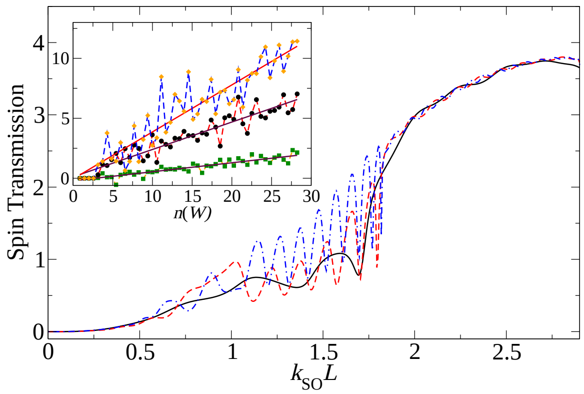

FIG. 3: (*Color online*) Ratchet spin transmission, Eq. (11), as a function of Rashba spin-orbit interaction $k_{\mathrm{SO}}L$ for different numbers of potential barriers: $N = 20$: full line, $N = 40$: dashed (red), and $N = 60$: dashed-dotted (blue). The energy is $E_{\mathrm{F}} = 36$. The width W is chosen such that $n(W) = 8$ open transverse modes exist. Inset: Ratchet spin transmission as a function of open modes for fixed $E_{\mathrm{F}} = 36$ and $N = 40$. Diamonds correspond to $k_{\mathrm{SO}}L = 2.5$, circles to $k_{\mathrm{SO}}L = 1.5$ and squares to $k_{\mathrm{SO}}L = 0.7$.

(solid line, Eq. (11)), T_{++} (dashed) and T_{--} (dashed-dotted) as a function of the dimensionless Rashba SO interaction $k_{\mathrm{SO}}L = \pi L/L_{\mathrm{SO}}$, in terms of the spin-precession length L_{SO}. For InAs quantum wells, L_{SO} ranges from 0.2 to 1 μm [23], for GaAs, L_{SO} can be tuned up to 4.4 μm [24]. Panels (a) and (b) show representative results for two values of E_{F} (vertical lines a and b in Fig. 1). Panel (a) corresponds to an energy, where the mode $n = 1$ (with remaining energy in longitudinal direction closely above U_0) couples to $n = 2$ (with Bloch band-type resonant transport). This Bloch-band mediated spin transmission leads to a negative spin current. The peak structure in the transmission curves in Fig. 2(a) results from SO-affected shifts of the resonant energy levels in the 5-barrier system. At energies of Fig. 2(b) mode $n = 1$ (above barrier propagation) couples to $n = 2$ (off-resonant tunneling transmission in the "band gap"). Here one finds, on the whole, a growing, positive spin transmission with increasing SO coupling [25].

For systems with bulk inversion asymmetry, such as GaAs, Dresselhaus SO interaction [10] has also to be considered. To this end we add a term $\mathcal{H}_{\mathrm{SO}}^{\mathrm{D}} = (\hbar k_{\mathrm{SO}}^{\mathrm{D}}/m^*)(\sigma_x p_x - \sigma_z p_z)$ to the Hamiltonian (1). The two insets in Fig. 2 demonstrate how the ratchet spin transmissions change for various values of $k_{\mathrm{SO}}^{\mathrm{D}}L$. In particular, we find spin current reversals at the symmetry points $k_{\mathrm{SO}} = k_{\mathrm{SO}}^{\mathrm{D}}$, where the SO effects cancel [26].

In Fig. 3 we summarize our main results for the spin transmission (11) of ratchet setups approaching the limit of extended periodic systems and many transverse modes. There the spin transmission arises from the complicated coupling of various modes and different Bloch bands. In Fig. 3 is shown the spin transmission as a function of $k_{\mathrm{SO}}L$ for different barrier numbers N, but fixed Fermi energy and width W such that the number of conducting modes is $n(W) = 8$. With increasing N a pronounced oscillation pattern appears arising from commensurability effects between L_{SO} and L [21]. More interestingly, as depicted in the inset, the ratchet spin transmission grows *linearly* with the number $n(W)$ of available modes with a slope depending on the SO coupling. Hence, the normalized spin transmission $2T_{\mathrm{S}}/T_{\mathrm{C}}$ (Eqs. (8,11)) is independent of $n(W)$ for the range of widths considered.

To summarize, we demonstrated that ratchets built from mesoscopic conductors with SO interaction generate reasonable, spin currents in an experimentally accessible parameter regime. Many further interesting questions open up within this new concept, including the exploration of spin ratchet effects for dissipative, diffusive and non-equilibrium particle and spin dynamics.

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